

ENG H191: Hands on Labs

Material Joining and Beam Bending

Engineering Disciplines Explored: Materials, Mechanical, Civil, Welding

MOTIVATION

Engineering Applications

Choosing suitable materials is important for all engineers as they design systems and products, whether those materials are semiconductors, polymers, or alloys. Engineers must take into account the overall goals and requirements of a system when making these choices. The study of the material properties allows engineers to predict how different materials will perform in a specific application and to choose a material that is most appropriate for their system.

When dealing with civil and mechanical applications, engineers are often interested in properties that characterize how a material behaves under load. One such property is elasticity, or a material's resistance to change in shape. The relative weight, or density, of a material may also be a consideration. Also, as is the case with all engineering, the cost of a material is usually important.

Objectives

- Demonstrate an understanding of material elasticity
- Describe how elasticity and geometry effect the strength of a beam
- Demonstrate ability to weld PVC

Basic Principles

This lab write-up will cover the following basic principles:

- Stress and strain
- Young's modulus (elasticity)
- Cantilever beams
- Deflection
- Welding plastics

Lab Experience

The lab experience will include observation and experimentation with:

- Measuring geometry of beams to calculate the area moment of inertia
- Measuring deflection of cantilever beams under various loads
- Experimentally determining Young's modulus for different materials
- Comparison of strength for beams with different geometries and materials
- Welding plastics

Fundamentals

Stress and Strain

Consider a beam loaded with a compressive force, as shown in Figure 1(a). Assuming that the system is in equilibrium (no acceleration), we know from Newton's third law that the forces applied to the end of the beam must be opposed by some equal and opposite force. These opposing forces (the dashed arrows in the figure) occur within the beam and are the result of the material within the beam resisting deformation. The distribution of these internal forces is called *stress* (denoted by σ). Stress is measured as force per unit of cross-sectional area and can be thought of as a kind of internal pressure. The SI unit for stress is N/m^2 , or Pascals (Pa).

If the forces applied to the beam are great enough, the beam will deform, or change shape. In Figure 1(a), the applied forces on top of the beam are causing the beam to shorten from its original length (shown in Figure 1(b)). We can measure the *deflection*, or change in length, due to some applied force. *Strain* (denoted by ϵ) is then the deflection per unit length (of the unloaded beam). Since strain is a length per unit length, it is a unit-less quantity.

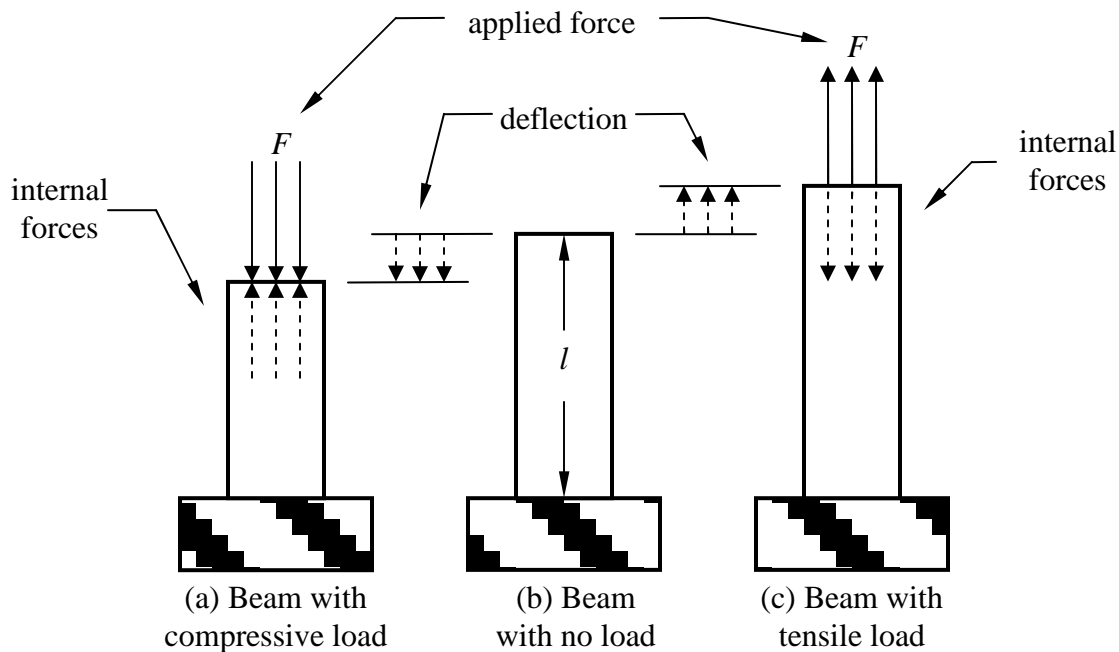


Figure 1: Pictorial of a beam under no load (b) and the same beam under compressive (a) and tensile (c) load.

The concepts of stress and strain also apply to systems under tensile loads. Figure 1(c) shows a beam that is being stretched by an external force. Again, there are internal forces within the beam that are “trying” to return the beam to its unloaded length. The distribution of these forces per unit area is the *stress* on the beam, and the amount of deflection divided by the original length is the *strain*.

Young's Modulus (Modulus of elasticity)

When examining the properties of materials, it is useful to find measurable quantities that do not depend on the size or shape of an object. This allows engineers to apply these quantities to predict the behavior of the material in many different applications, from building airplanes to designing aluminum cans.

It turns out that the interaction between stress and strain gives one such quantity, known as Young's modulus, or the modulus of elasticity, E , given by the equation:

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon} \quad \text{with units} \quad \frac{N/m^2}{m/m} = N/m^2 = Pa$$

Note that Young's modulus does not depend on the length of an object. Furthermore, since it is a force *per* unit area it doesn't depend on the cross-sectional area either. Thus Young's modulus is not dependent on the geometry of the object, but only on the internal properties of the material. We can use Young's modulus to predict how an object of any size or shape will deform under a given load.

Cantilever Beams

A cantilevered beam is generally a horizontal beam with one end securely fastened and the other end hanging freely as shown in Figure 2. Most often the purpose of the beam is to support a vertical load and, depending on the application, it should do this without deflecting too much. Another application may be to use a cantilever beam to act as a spring, either storing energy or to achieve a desired force-deflection relationship.

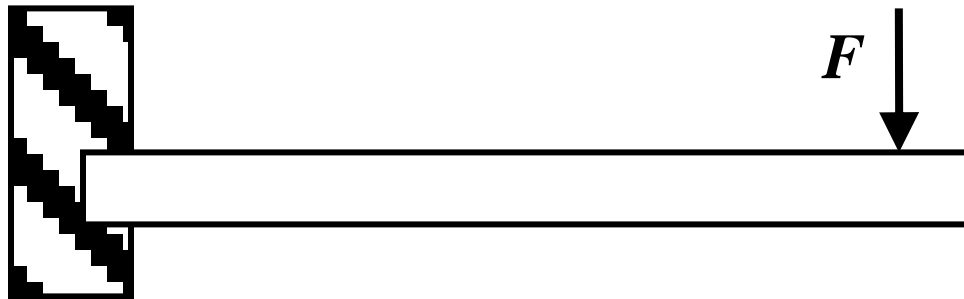


Figure 2: A cantilevered beam with applied load

Stress and Strain in a Cantilever Beam

In Figure 1, we relate stress and strain to a beam that is being compressed and stretched. For the case of cantilevered beams, the deformation is slightly more complicated because the applied force will cause the beam to bend. However, we can think of bending in terms of compression and stretching at the same time.

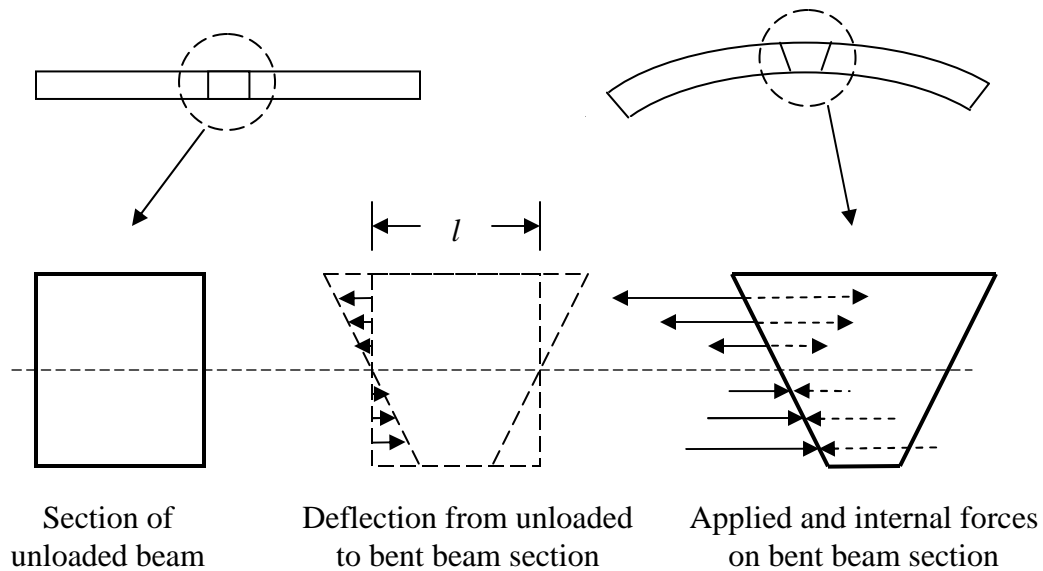


Figure 3: Pictorial showing the internal forces and resulting deflection of a section of bent beam

Figure 3 shows a bent beam on the right and the same beam with no load on the left. As the beam is bent, the upper surface becomes longer and the lower surface becomes shorter, while the centerline remains the same length. This becomes evident when we compare a section of the unbent beam to the same section of the bent beam, as shown in the figure. So the deflection of the beam, and thus also the *strain*, increases with the distance from the centerline. Similarly, the forces acting on the section of the bent beam are strongest at the upper and lower surfaces and nearly zero at the centerline. This means that the opposing internal forces, and so the *stress*, also increase with the distance from the centerline.

Because stress and strain within the bent beam are not constant across its cross-section, it becomes difficult to develop equations involving the deflection. However, it is important to note that the stress and strain are proportional to one another at all distances from the centerline, and the constant of proportionality is Young's modulus.

Predicting deflection of a cantilevered beam

For a cantilever beam, we measure the deflection as the amount that the beam bends due to the load placed on it. This deflection depends on:

- The load: more deflection with larger load.
- The length of the beam: the deflection increases with the length.
- The Young's modulus: higher E produces less deflection.
- The geometry of the cross section: higher area moment of inertia results in less deflection.

The deflection of a cantilevered beam at the distance, s , from the clamped end may be predicted by the following equation. An illustration of the variables of interest is given in Figure 4. Note that deflection is positive upwards, while force is positive downwards.

$$\delta = -\frac{Fs^2}{6EI}(3L - s)$$

where:

L = distance to load

s = distance to dial indicator

F = load

δ = deflection at point of dial indicator

E = elasticity (Young's) modulus

I = area moment of inertia of the cross section

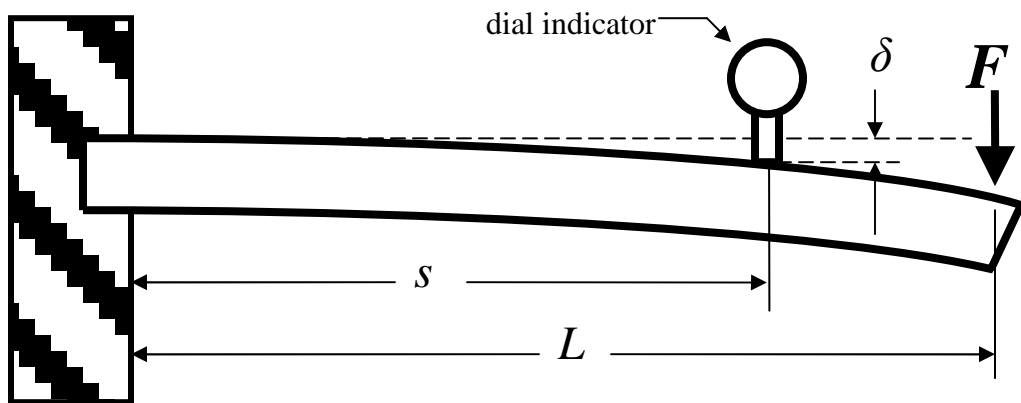


Figure 4: Predicting the deflection of a cantilevered beam under load

Area Moment of Inertia

Area moment of inertia of a beam's cross-section is related to the beam's resistance to bending. The quantity is related solely to the cross-sectional geometry and the reference axis, in this case the centerline. Area moment can be computed for any 2-D shape using the integral of the squared distance of each area element from the reference axis:

$$I_x = \int_A x^2 dA$$

where:

x = distance to reference axis

A = area

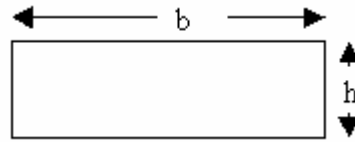
I = area moment of inertia of the cross section

Fortunately, for many common shapes, the area moment of inertia can be reduced to an algebraic expression that can be found in a table. For the shapes explored in this lab these expressions are given below.

1. Rectangular:

The formula for the area moment of inertia for a rectangle cross section is

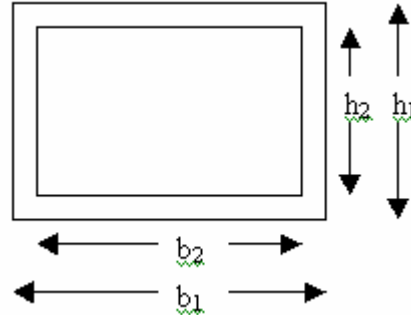
$$I = \frac{b_1 h_1^3}{12}$$



2. Box:

The formula for the area moment of inertia for a bar with a box cross section is

$$I = \frac{b_1 h_1^3 - b_2 h_2^3}{12}$$



Welding Plastics

Welding is a process by which two members are permanently joined, usually by heating the joint to allow the two materials to bond. Welding is an alternative to other fasteners such as nuts and bolts. The advantages of welding are that it can be cheaper and provides a stiffer joint. Disadvantages are that the process may warp the materials and the assembly can not be taken apart.

Lab Experience

Make sketches of set-ups used in class; include them in your lab report.

Beam Bending

In the lab, three cantilever beams will be set up in order to compare their deflection. Two of the beams have the same cross sectional geometry (rectangle), but one is made of steel and the other is made of aluminum. The third beam is made of aluminum with a smaller cross-sectional area than the first two beams, but with a box shaped cross-section.

To do the bending tests, you will clamp the beams to the bench top and measure deflection while you apply known loads. By clamping the beam to the bench you are creating a cantilever beam. You will use a dial indicator to measure the deflection.

For each of the three beams (steel rectangular, aluminum rectangular, and aluminum box):

1. Clamp beam and position the dial indicator such that $L = 12.5$ in. and $s = 11.5$ in. (see Figure 4) *** Be gentle with the dial indicator!
2. Load (by placing 2.5 lb. weights in the bucket) incrementally up to $F_{\max} = 12.5$ lbs. *** Place weights SLOWLY. Do not drop.
3. Measure the beam's cross-sectional dimensions and compute area moment of inertia, I . Record into Data Table 1 (included at the end of this document).
4. Record the deflection, δ , for each load using Data Tables 2-4.

Plastic Welding

In the spring quarter, the teams building robots will be given the option of using welded PVC (Polyvinyl Chloride) as a building material in addition to such options as steel Erector set parts which are assembled with nuts and bolts. In this laboratory, your team will be introduced to working with the PVC material to gain some experience. Teams will actually try welding PVC using a welding torch, a PVC welding rod and PVC parts. The temperatures are high and you will have to exercise caution.

1. Read the handout "Plastic Welding" before using the welding equipment.
PLEASE NOTE THAT THE METAL PART OF THE WELDING TORCH IS EXTREMELY HOT AND MAY CAUSE SERIOUS INJURIES IF TOUCHED.
2. Weld the two PVC beams provided in order to create a "T" bar as shown in Figure 5 (next page).
3. Sketch the test setup and write down your observations.
Note: All teammates will try to weld on the same structure.

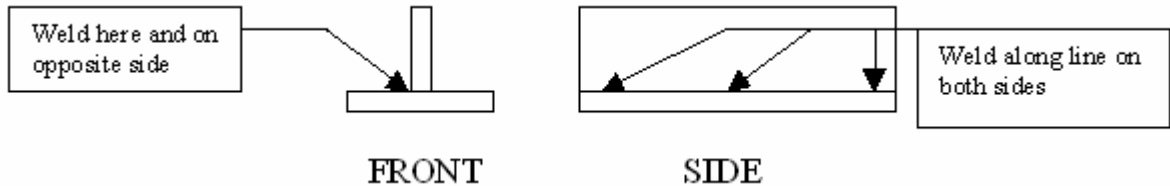


Figure 5: Creating a PVC weld

Questions

- Compare theoretical to experimental deflection and discuss any differences.
- Describe uses for steel vs. aluminum based on these results.
- What are the critical factors for obtaining a good weld?
- What were the problems encountered in welding, and what were the results of those problems?
- Which materials and methods of assembly would be suitable for building a small (9 inch by 9 inch robot)? Discuss the benefits and drawbacks for the materials already explored (welded PVC and Erector Set (recall Lab 1)) as well as any others you might have in mind.

Lab Report

Individual lab reports. The rubric will not contain details about lab-specific requirements. It is up to you to determine what content is appropriate based on the experiment and on this write-up. Some hints:

- GENERAL – Be sure to include all required sections and content.
- EXPERIMENTAL METHODOLOGY – Make sure your reader has a good understanding of the experimental set-up and your method for taking measurements.
- RESULTS – Think about how to best present your data. What did you measure, and what are you going to compare it to? How can you represent this data so that your reader can easily see and understand it?
- DISCUSSION – Be sure that your discussion is objective. Avoid vague language; quantify observations whenever possible. The questions listed above will guide some of your discussion section. For additional material, look for other topics introduced in this write-up or in the lecture. It is always a good idea to address the accuracy of the experiment and its results if you think there might have been any problems.

Data Tables

	E, Modulus of Elasticity (psi)	Width, b (in.)	Height, h (in.)	Wall Thickness (in.)	I, Moment Of Inertia (in ⁴)
Steel Rectangular	29x10 ⁶			N.A.	
Aluminum Rectangular	10.1x10 ⁶			N.A.	
Aluminum Box	10.1x10 ⁶				

Table 1: Beam Dimensions

Load F, (lb)					
Deflection δ (in.)					

Table 2: Steel Rectangular Beam

Load F, (lb)					
Deflection δ (in.)					

Table 3: Aluminum Rectangular Beam

Load F, (lb)					
Deflection δ (in.)					

Table 4: Aluminum Beam with Box Cross-Section

Notes: